Q26 Solve the difference equation by z-transform

$$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n)$$

with the response $y(n)$ to the input $x(n) = \delta(n)$ if initial conditions are $y(-1) = -1$, and $y(-2) = 1$

$$Y(z) = \frac{3}{4}\left[z^{-1}Y(z) + y(-1)\right] - \frac{1}{8}\left[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)\right] + X(z)$$

We have $x(n) = \delta(n)$

$$X(z) = 1$$

$$Y(z) = \frac{3}{4}\left[z^{-1}Y(z) - 1\right] - \frac{1}{8}\left[z^{-2}Y(z) - z^{-1} + 1\right] + 1$$

$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = -\frac{3}{4} + \frac{1}{8}z^{-1} - \frac{1}{8} + 1$$

$$Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = -\frac{6-1+8}{4} + \frac{1}{8}z^{-1}$$

$$Y(z)\left[\frac{8 - 6z^{-1} + z^{-2}}{8}\right] = \frac{1}{4} + \frac{1}{8}z^{-1}$$

$$Y(z) = \frac{8z}{4(8 - 6z^{-1} + z^{-2})} + \frac{8z^{-1}}{8(8 - 6z^{-1} + z^{-2})}$$

$$Y(z) = \frac{2}{8 - 6z^{-1} + z^{-2}} + \frac{z^{-1}}{8 - 6z^{-1} + z^{-2}}$$

$$Y(z) = \frac{2}{(1-z^{-1})(1+z^{-1})}$$

$$Y(z) = \frac{2z^2}{z^2-6z+8} + \frac{z}{z^2-6z+8}$$

$$= \frac{2z^2}{(\quad)(\quad)}$$

$$= \frac{2z^2+z}{z^2-6z+8}$$

$$= \frac{2z^2+z}{(z-2)(z-4)}$$

$$A = (z-2)\left.\frac{2z^2+z}{(z-2)(z-4)}\right|_{z=2}$$

$$= \frac{8+2}{-2} = -5$$

$$B = (z-4)\left.\frac{2z^2+4}{(z-2)(z-4)}\right|_{z=4}$$

$$= \frac{32+4}{2} = \frac{36}{2} = 18$$

$$\frac{-5}{z-2} + \frac{18}{z-4}$$

$$\frac{-5z^{-1}}{1-2z^{-1}} + \frac{18z^{-1}}{1-4z^{-1}} = -5(2)^n u(n-1) + 18(4)^n u(n-1)$$